

Przeznaczam się z trzema podkreśleniami na stronie 219.

Przeznaczam się ćwic 2, ćwic 3 i ćwic 4 ze str. 219.
b) f) b) f) b) f)

Poniżej przedstawiam przykładowy sposób rozwiązywania:

Ćw 2

e) $a_1 = 1, a_2 = -2, a_3 = 4$

Zauważamy, że $q = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots$ zatem $q = \frac{-2}{1} = -2$

$$a_4 = a_3 \cdot q = 4 \cdot (-2) = -8$$

$$a_5 = a_4 \cdot q = -8 \cdot (-2) = +16$$

Ćw 3

e) $a_1 = \frac{1}{\sqrt{2}}, q = \sqrt{2}$

$$a_2 = a_1 \cdot q = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$$

$$a_3 = a_2 \cdot q = 1 \cdot \sqrt{2} = \sqrt{2}$$

$$a_4 = a_3 \cdot q = \sqrt{2} \cdot \sqrt{2} = 2$$

$$a_5 = a_4 \cdot q = 2 \cdot \sqrt{2} = 2\sqrt{2}$$

Ćw 4

e) $a_{m-1} = \sqrt{2}$ i $a_m = -4$, zatem $q = \frac{a_m}{a_{m-1}} = \frac{-4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-4\sqrt{2}}{2} = -2\sqrt{2}$

c) $a_{m-1} = -216, a_m = -36$, $q = \frac{-36}{-216} = \frac{1}{6}$

Ćw 5 s 220

b) sprawdzamy, czy

$$\frac{3}{8} \div \frac{1}{2} = \frac{3}{16} \div \frac{3}{8} = \frac{27}{32} \div \frac{8}{16} = \frac{81}{64} \div \frac{27}{32}$$

$\frac{3}{8} \div \frac{1}{2} = \frac{3}{8} \cdot \frac{2}{1} = \frac{6}{8} = \frac{3}{4}$
 $\frac{3}{16} \div \frac{3}{8} = \frac{3}{16} \cdot \frac{8}{3} = \frac{24}{48} = \frac{1}{2}$
 $\frac{27}{32} \div \frac{8}{16} = \frac{27}{32} \cdot \frac{16}{8} = \frac{27}{16}$
 $\frac{81}{64} \div \frac{27}{32} = \frac{81}{64} \cdot \frac{32}{27} = \frac{27}{16}$

Zatem nie jest to ciąg geometryczny

Proszę wykonać punkt a i b s 220 w analogiczny sposób. -2-

Ćw 6

g) $a_1 = 5$ $q = \sqrt{2}$ $n = 9$

$$a_n = a_1 \cdot q^{n-1}$$

$$a_9 = a_1 \cdot q^{9-1} = 5 \cdot (\sqrt{2})^8 = 5 \cdot (2^{\frac{1}{2}})^8 = 5 \cdot 2^{\frac{8}{2}} = 5 \cdot 2^4 = 5 \cdot 16$$

$$\underline{a_9 = 80}$$

d) $a_1 = \frac{1}{10}$ $q = 2^{\frac{1}{2}} = \frac{\sqrt{2}}{2}$, $n = 6$

$$a_6 = a_1 \cdot q^5 = \frac{1}{10} \cdot \left(\frac{\sqrt{2}}{2}\right)^5 = \frac{1}{10} \cdot \frac{5^4}{2^5} = \frac{5^4}{2^6} = \frac{625}{64}$$

Proszę wykonać punkty a) i f) Ćw 6 s 220.

Ćw 7 s 220

c) $a_8 = 81$, $q = 3$ Szukane: $a_1 = ?$

zauważmy, że jeśli $a_n = a_1 \cdot q^{n-1}$ to $a_8 = a_1 \cdot q^7$ zatem $81 = a_1 \cdot 3^7$

$$\text{stąd } a_1 = \frac{81}{3^7} = \frac{3^4}{3^7} = \frac{1}{3^3} = \frac{1}{27}$$

Proszę wykonać punkt b) Ćw 7 s 220

Ład 1 s 220

e) $a_1 = \sqrt{2}$, $a_2 = -2$, $a_3 = 2\sqrt{2}$, ...

$$q = \frac{a_2}{a_1} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = \underline{\underline{-\sqrt{2}}}$$

$$a_n = a_1 \cdot q^{n-1} = \sqrt{2} \cdot (-\sqrt{2})^{n-1} = (\sqrt{2})^n \cdot (-1)^{n-1} \cdot (\sqrt{2})^{n-1} = (-1)^{n-1} \cdot (\sqrt{2})^n = (-1)^{n-1} \cdot 2^{\frac{n}{2}}$$

$$a_7 = (-1)^{7-1} \cdot 2^{\frac{7}{2}} = (-1)^6 \cdot 2^{3\frac{1}{2}} = 2^3 \cdot 2^{\frac{1}{2}} = 8\sqrt{2}$$

c) $a_1 = \frac{1}{243}$ $a_2 = -\frac{1}{81}$ $a_3 = \frac{1}{27}$

$$q = \frac{a_2}{a_1} = \left(-\frac{1}{81}\right) : \frac{1}{243} = -\frac{1}{81} \cdot \frac{243}{1} = \underline{\underline{-3}}$$

$$a_n = \frac{1}{243} \cdot (-3)^{n-1} = \frac{(-3)^{n-1}}{3^5} = (-1)^{n-1} \cdot \frac{3^{n-1}}{3^5} = (-1)^{n-1} \cdot 3^{n-1-5} = (-1)^{n-1} \cdot 3^{n-6}$$

$$a_7 = (-1)^{7-6} \cdot 3^{7-6} = 1 \cdot 3^1 = \underline{\underline{3}}$$

Zad 2 s 221

$$b) a_1 = \sqrt{2} - 1, a_2 = 1, a_3 = \sqrt{2} + 1, \dots$$

Szukane: $a_4 = ?$, $a_5 = ?$

zakładamy, że $q = \frac{a_2}{a_1} = \frac{1}{\sqrt{2}-1}$ (możliwość ułamka niewymierność z mianownikami)

$$q = \frac{1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{\sqrt{2}^2 - 1^2} = \frac{\sqrt{2}+1}{2-1} = \frac{\sqrt{2}+1}{1} = \underline{\underline{\sqrt{2}+1 = q}}$$

$$a_4 = a_3 \cdot q = (\sqrt{2}+1) \cdot (\sqrt{2}+1) = \sqrt{2}^2 + 2\sqrt{2} + 1^2 = 2 + 2\sqrt{2} + 1 = \underline{\underline{3 + 2\sqrt{2}}}$$

$$(\sqrt{2}+1)^2 = a^2 + 2 \cdot a \cdot b + b^2$$

$$a_5 = a_4 \cdot q = (3 + 2\sqrt{2})(\sqrt{2}+1) = 3\sqrt{2} + 3 + 2\sqrt{2} \cdot \sqrt{2} + 2\sqrt{2} \cdot 1 = 3\sqrt{2} + 3 + 4 + 2\sqrt{2} = \underline{\underline{5\sqrt{2} + 7}}$$

Proszę audytorów wykonać przykład a) zad. 2 s 221.

Zad. 3

Proszę zapisać się z przykładem reprezentacyjnym w zadaniu, a następnie wykonać przykłady d) i e).

$$a) a_2 = \frac{5}{12}, a_3 = \frac{2}{15} \quad \begin{cases} a_2 = a_1 \cdot q \\ a_3 = a_1 \cdot q^2 \end{cases} \Leftrightarrow \begin{cases} \frac{5}{12} = a_1 \cdot q \cdot 12 \\ \frac{2}{15} = a_1 \cdot q^2 \cdot 15 \end{cases} \quad \begin{cases} 5 = 12 a_1 q \\ 2 = 15 a_1 q^2 \end{cases} \Leftrightarrow$$

$$\frac{5}{2} = \frac{12}{15q} \Leftrightarrow 5 \cdot 15q = 2 \cdot 12 \Leftrightarrow 75q = 24 \quad | : 5 \quad q = \frac{24}{75} = \frac{8}{25}, \text{ jeśli } a_2 = a_1 q$$

$$a_1 = \frac{a_2}{q} = \frac{5}{12} : \frac{8}{25} = \frac{5}{12} \cdot \frac{25}{8} = \underline{\underline{\frac{125}{96}}}$$

$$b) a_3 = 1\frac{1}{2} = \frac{3}{2}, a_4 = 1\frac{1}{4} = \frac{5}{4}, q = \frac{a_4}{a_3} = \frac{5}{4} : \frac{3}{2} = \frac{5}{4} \cdot \frac{2}{3} = \frac{5}{6}$$

$$a_1 = \frac{a_2}{q} = \frac{9}{5} : \frac{5}{6} = \frac{9}{5} \cdot \frac{6}{5} = \underline{\underline{\frac{54}{25}}}$$

$$a_2 = \frac{a_3}{q} = \frac{3}{2} : \frac{5}{6} = \frac{3}{2} \cdot \frac{6}{5} = \underline{\underline{\frac{9}{5}}}$$

-4-

$$q = \frac{a_6}{a_5} = \frac{5}{3} \quad \text{or} \quad a_4 = \frac{a_5}{q} = 3 \cdot \frac{5}{3} = 3 \cdot \frac{3}{5} = \frac{9}{5}$$

$$a_3 = \frac{a_4}{q} = \frac{9}{5} \cdot \frac{5}{3} = \frac{9}{5} \cdot \frac{3}{5} = \frac{27}{25}$$

$$q_2 = \frac{q_3}{q_1} = \frac{24}{25} \cdot \frac{5}{3} = \frac{24 \cdot 3}{25 \cdot 5} = \frac{81}{125}$$

$$a_1 = \frac{a_2}{q} = \frac{81}{125} \cdot \frac{5}{3} = \frac{81}{125} \cdot \frac{3}{5} = \frac{243}{625}$$

$$a_m = a_1 q^{m-1} \text{ Wtedy:}$$

$$a_3 = 3 \Leftrightarrow a_3 = a_1 \cdot q^2 \quad 3 = a_1 \cdot q^2 \quad \text{oder} \quad a_6 = a_3 \cdot q^3 = -81 \quad \text{steht}$$

$$-81 = 3 \cdot q^3 \quad | :3 \quad q^3 = -27 \quad (\Rightarrow) \quad \underline{q = -3} \quad \text{oder} \quad 3 = a_1 \cdot (-3)^2 \quad \Leftrightarrow \quad 3 = 9a_1 \quad | :9$$

cykli' $\underline{a_1 = \frac{1}{3}}$, $\underline{a_2 = a_1 q = \frac{1}{3} \cdot (-3) = -1}$, $\underline{a_4 = a_3 q = 3 \cdot (-3) = -9}$, $\underline{a_5 = a_4 q = -9 \cdot (-3) = 27}$

b) $x=3, y=768$

$$\begin{array}{ccccccc} X & ? & ? & ? & & & y \\ \parallel & \parallel & \parallel & \parallel & \parallel & \parallel & \parallel \\ 3 & 3 \cdot q & 3 \cdot q^2 & 3 \cdot q^3 & & & 468 \\ \parallel & \parallel & \parallel & \parallel & \parallel & \parallel & \parallel \\ a_1 & a_2 & a_3 & a_4 & & & 3 \cdot q^4 \\ & & & & & & \parallel \\ & & & & & & a_5 \end{array}$$

stgd $768 = 3q^4 \mid :3$

$$9^4 = 256$$

zauważamy, że
 $q^4 = 4^4$ lub $q^4 = (-4)^4$

Steady $q = 4$ lub $q = -4$

yes! $q=4$, to $a_2 = 3 \cdot 4 = 12$, $a_3 = a_2 \cdot q = 12 \cdot 4 = 48$, $a_4 = a_3 \cdot q = 48 \cdot 4 = 192$

yes! $a = -4$ to $a_2 = 3 \cdot (-4) = -12$, $a_3 = -12 \cdot (-4) = 48$, $a_4 = 48 \cdot (-4) = -192$

Proszę, androp'ane wykonać pyłkiad a) i c).